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Creation of a generalized dynamic model of planetary moons based on an analytical approach for describing the libration processes of their rotation

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Abstract. Our work is devoted to the creation of a generalized dynamic model that describes libration processes in the motion of the natural satellite of the planet, which has been tested on the simulation of the moons rotational parameters(MRPs) of the Earth's Moon and allows adapting description system of MRPs to other celestial objects, having a librational nature of rotation as well.In this case, it is assumed that an analytical approach was used to describe the rotational motion, which, on the one hand, is rather complicated for solving the equations of rotation and has a lower accuracy than the numerical one, but, on the other hand, the resulting analytical representation of MRPs provides a more convenient tool for analyzing the behavior of MRPs with changes in various parameters, determining the rotational dynamics of the satellite.It is with its help that the procedure of flexible computer simulation of the rotation process and the identification of those observational manifestations, which are determined primarily by the parameters of the figure of a celestial body - the laws of distribution of body mass, compression, non-sphericity.

Keywords: physical libration, dynamic model of the moon, planets

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Материалы конференции
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Создание обобщенной динамической модели естественных спутников планет на основе аналитического подхода для описания либрационных процессов их вращения

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Аннотация. Работа посвящена созданию обобщенной динамической модели, описывающей либрационные процессы в движении естественного спутника планеты, которая апробирована на моделировании ПВС естественного спутника Земли и позволяет адаптировать систему описания ПВС и на другие небесные объекты, имеющие либрационный характер вращения.

Ключевые слова: физическая либрация, динамическая модель Луны, планеты

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Introduction

Natural moons systems represent a wide variety of orbital-rotational configurations in the Solar system. [1]. While some of these are clearly the result of tidal processes, others still have largely unknown moons rotational parameters (MRPs) and structural models [1]. At the same time, the knowledge of MRPs makes it possible to study the internal structure of these bodies without expensive, and most often, simply inaccessible space experiments on the lunar surface or its nearby area. Observations of the rotation of the moons of Jupiter, Saturn and the Earth [2–6] made it possible to make the first estimates of their internal structure, the distribution of heat fluxes, and even their chemical composition [7–9]. Of particular interest in this regard are those moons that have passed the stage of tidal evolution and are in stable rotation: so-called libration moons. The physical libration of the Moon (PLM) is the oscillation of a celestial body about the axis of rotation under the influence of perturbing forces. So, for the Moon, when constructing the most accurate theory of libration, it is necessary to take into account the influence of the gravity of the Sun and the Earth on the dynamics of the Moon. For other moons, the influence of the Sun on rotation can most likely be neglected due to the large distance from it. But at the same time, accounting for the central planet, such as Jupiter and Saturn, becomes more significant. Since the influence of the central planet on the moons is very strong, almost all of them have passed the stage of tidal evolution, have a figure strongly elongated towards the planet, and are in stable rotation. Like the Moon, the Galilean moons of Jupiter make one orbit in a period equal to one revolution around the axis. This phenomenon is called orbital - rotational resonance and for these moons is 1:1.

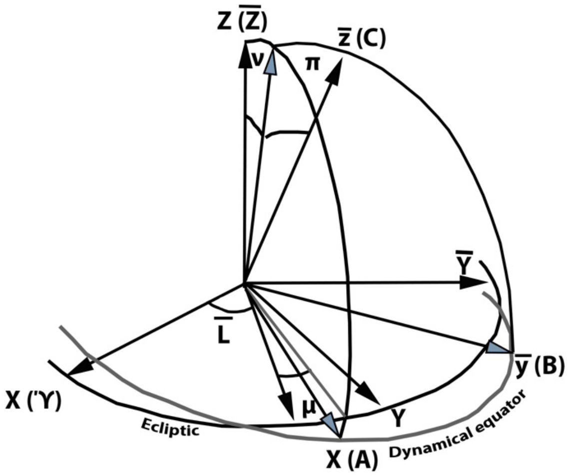

Fig. 1. Selenocentric coordinate systems. *XYZ* is the ecliptic coordinate system. *X*-axis direct to ♈ (the vernal equinox), *Z*-axis direct to the ecliptic pole. $\left(\bar{X},\bar{Y},\bar{Z}\right)$ is a uniformly rotating ecliptic coordinate system

In this paper, we presented a strategy for solving the physical libration theory of a satellite (PLS) problem based on the method of constructing the theory of the PLM. In practice, for any moon, including the Moon, an important simplification can be introduced: orbital movement does not affect the rotation of the moon in any way.

Materials and Methods

At the beginning, it was believed that the motion of a celestial body in orbit is described by Newton's second law. However, over time, Lagrange, and then Hamilton, Jacobi and other researchers modified Newton's second law so that the algorithm for solving such a problem would consist of certain stages. In this work, we used the approach proposed by Hamilton. As a result, a system of Hamiltonian differential equations of the first order was obtained, solving which, we obtain the parameters of the PLM theory.

It should be noted that the canonical variables are the transition angles from the inertial coordinate system to the dynamic one (DSC) tied to the principal axes of inertia. In other words, we can say that these are the transition angles from the absolute coordinate system to the rotating one. We have chosen the aircraft angles as such angles, which determine the position of the DSC relative to the ecliptic. The geometric meaning of these angles is described below and shown in Fig. 1. The algorithm for such a transition is as follows:

Rotation around the pole of the ecliptic Z by an angle, $M = \overline{L} + \mu$ from the X axis to the average direction to the Earth $x(A)$;

Rotation around the $\overline{Y}$ axis by an angle ν, until it coincides with the $x(A)$ axis;

Rotation around the $x(A)$ axis by an angle π, until it coincides with the $y(B)$ axis.

This algorithm can be written using rotation matrices:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \prod_{x}(-\pi) \times \prod_{\overline{Y}}(\nu) \times \prod_{\overline{Z}}(L+\mu) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{1}$$

It is these libration angles that we have taken as canonical angular variables

$$q_1 = \mu,\ q_2 = \nu, q_3 = \pi.$$

The conjugate canonical moment we define as

$$p_1 = \frac{\partial T}{\partial q_1},\ p_2 = \frac{\partial T}{\partial q_2},\ p_3 = \frac{\partial T}{\partial q_3}.$$

Here T is the kinetic energy of the Moon's rotation, the derivation of the expression for which we describe below.

The expression for the kinetic energy can be written in terms of the Moon's inertia tensor as follows:

$$\mathbf{G} = J\omega = \begin{pmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{pmatrix}. \tag{2}$$

Then the kinetic energy of the system will have the form:

$$T = \frac{1}{2}\omega^T J\omega. \tag{3}$$

For the working formula of kinetic energy, it is necessary to determine the projections of the angular velocity for a rotating coordinate system. This is done using simple geometric transformations:

$$\Omega_x = -\dot{M} \times \sin \nu - \dot{\pi},$$
$$q = \Omega_y = -\dot{M} \times \cos \nu \times \sin \pi + \dot{\nu} \times \cos \pi, \quad (4)$$
$$q = \Omega_y = -\dot{M} \times \cos \nu \times \sin \pi + \dot{\nu} \times \cos \pi.$$

After simple transformations, an expression for the kinetic energy is obtained, in which there are no components of the derivatives of the canonical parameters:

$$T = \frac{1}{2}[(1+k_2) \times p_3^2 +$$
$$+(1+k_1)(1+k) \times (p_2 \cos(q_3) - \sec(q_2)(n + p_1 - p_3)\sin(q_3))^2] + \quad (5)$$
$$+\frac{1}{2}[p_2 \times \sin(q_3) + \cos(q_3) + ((n+p_1)\sec(q_2) - p_3 \tan(q_2))]^2$$

where $k_1 = \frac{C-B}{B}$, $k_2 = \frac{C-A}{A}$ are expressed in terms of the main parameters of the Moon's inertia; p_1, p_2, p_3 are canonical impulses. Then the system of Hamilton equations in general form will look like this

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \ \frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}. \quad (6)$$

As a result, equations were obtained that describe the rotation of a celestial body, which can be brought into a general form to expand the gravitational potential of the moon:

$$U(\rho,\theta,\phi) = \frac{GM}{\rho}\left(\begin{array}{c} 1 + \left(\frac{a}{\rho}\right)^2 [c_{20}P_2(\cos\theta) + c_{22}\cos(2\lambda)P_{22}(\cos\theta)] + \\ +\sum_{n=3}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{\rho}\right)^n [c_{nm}\cos(m\lambda) + s_{nm}\sin(m\lambda)]P_{nm}(\cos\theta) \end{array} \right). \quad (7)$$

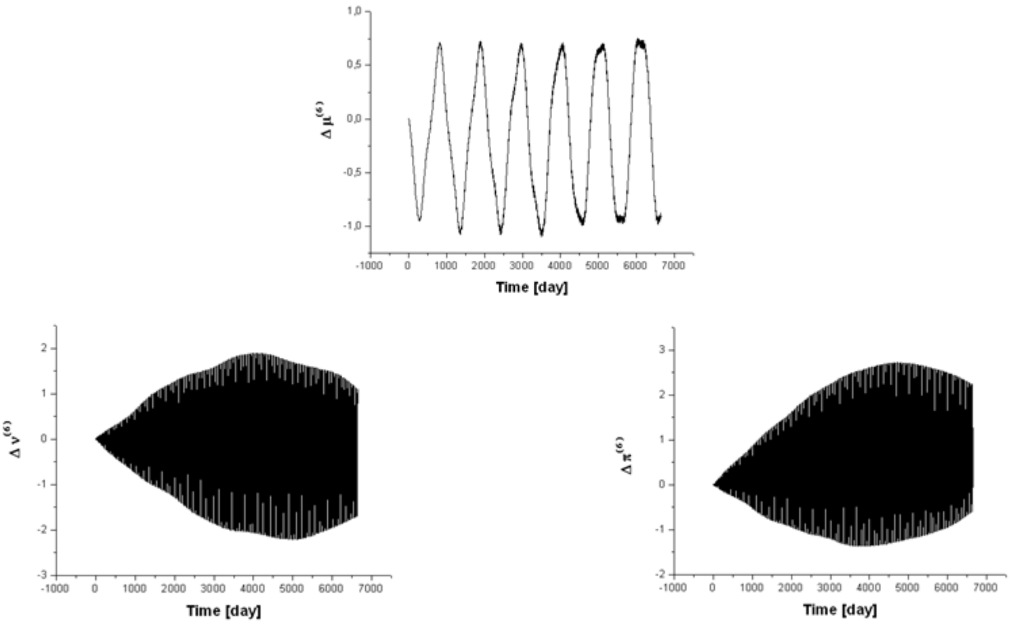

Fig. 2. Residual differences between numerical solution and dynamic ephemeris DE421

Results and Discussion

In order to assess the accuracy of the theory, a plot of residual differences with respect to the DE421 ephemeris was plotted (Fig. 2). The figure shows the residual differences in libration in longitude and latitude after taking into account all the perturbations included in our model.

In Fig. 2, the scale along the y-axis is expressed in arcseconds.It can be concluded that the amplitudes correspond to the order of one second for all libration angles, and this is a rather large value.If you perform a harmonic analysis of the results to identify the fundamental frequencies, then you get frequencies close to the eigenfrequencies of the system (1047 days, 27.3 days, 68 years). When compared with the theory (DE421), there are significant differences in the process of taking into account various perturbations, such as the lunar core, etc., and in the residual differences, this will manifest itself just at frequencies close to the natural ones.To increase the reliability of the PLM theory, it is possible to subtract the harmonics obtained for the Eigen frequencies of the system.

Conclusion

Consideration of perturbations from planets for natural satellites is carried out by switching to modern orbital theories of motion DE (dynamic ephemeris) developed at NASA's Jet Propulsion Laboratory, or to the EPM numerical theory developed at IAA RAS [10]. Naturally, the most accurate description of the rotational motion is made for the Moon [11]. If for the majority of moons it is possible to use only second-order harmonics in the expansion of the moon's gravitational field, then for the Moon there are good data for harmonics of the 3rd and higher orders.In our studies, we limited ourselves to the harmonics of the 4th order. In addition, laser ranging data made it possible to estimate the elastic properties of the lunar body [12].This gave us the opportunity to take into account elastic properties of the lunar body in the first approximation.

PLM is not only a tool for constructing a theory of oscillations of a celestial body about the axis of rotation, but also a method for modeling internal processes [13].Already a classic example of considering the process of rotation of a raw and liquid eggs allows us to say that such rotation is greatly influenced by the internal structure of the body itself [14].Therefore, our further research will be aimed at analyzing more subtle effects of the rotational motion of celestial bodies and deriving equations that take into account the presence of the lunar core [15].

The paper presents the result of many years of work on the construction of the theory of PLM, in which all dynamic constants and coefficients in the expansion of the potential were studied, and as a result, a mathematically rigorous theory of the rotation of a celestial body was constructed [16].The results of this work were reported at the All-Russian Astronomical Conference (VAK 2021), and were included in the list of the Scientific Council for Astronomy of the Russian Academy of Sciences as a significant contribution to astronomical research in 2021. All studies carried out in this work are new and performed using the methods developed by the authors. The results obtained can be used in the implementation of space missions to the bodies of the solar system and in the study of exoplanetary systems [17—20].

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