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Use of an Analytical Theory for the Physical Libration of the Moon to Detect Free Nutation of the Lunar Core

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Abstract—A brief review of modern observational achievements and the theoretical basis of physical libration of the Moon is presented. Special attention is given to the inferred existence of a lunar core and determination of its parameters. The creation of a theory of physical libration of the Moon, which requires analyses of semi-empirical series of long-term laser observations and the use of the highly accurate DE421 dynamical ephemeris, is related to this. A large role in this area has been played by the analytical theory of physical libration of the Moon constructed by Yu.V. Barkin, which made it possible for the first time to derive parameters of the free nutation of the lunar core from observations. This paper is based on a talk given at the conference "Modern Astrometry 2017," dedicated to the memory of K.V. Kuimov (Sternberg Astronomical Institute, Moscow State University, October 23–25, 2017).

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1. INTRODUCTION

The investigation of the internal structure of celestial bodies is a task facing scientists working in a variety of areas. A huge observational database has been accumulated for the Moon, including both traditional ground (heliometric and photographic) observations [1], seismic sounding using lunar surface modules [2, 3], theoretical and computer modeling based on the multi-faceted reduction of various types of data [4], and the most powerful source of information about the Moon—laser ranging using corner reflectors installed on the lunar surface in the 1960s [5–7].

Recent space missions and progress in studies of the lunar gravitational field have provided the conditions needed for the construction of highly accurate theories of the lunar rotation. For example, the Selene mission (Japan, 2007–2009) made it possible to appreciably refine the Stokes coefficients of orders two to four in expansions of the lunar potential, which most strongly influence the physical libration of the Moon (PhLM). Furthermore, new observational technologies in a system of three circumlunar satellites have enabled the refinement of the Love coefficient k_2, responsible for the visco-elastic properties of the lunar body [8, 9].

An improved model for the gravitational field based on data from the American GRAIL mission (2011–2012) [10, 11] is incorporated in the best numerical ephemeris of the Moon and the planets current available, DE430/431 [12], which provides submeter precision when compared with laser-ranging data.

All this variety of astronomical and geophysical methods has provided a reliable observational basis for studies of the structure of the Moon's body and its physical and chemical properties.

2. NUMERICAL AND ANALYTICAL APPROACHES IN THE THEORY OF PHYSICAL LIBRATION OF THE MOON

The results of theories of PhLM can be represented in various ways: as tables containing the results of numerical integrations or Poisson series determining the analytical dependences on time and the parameters of the dynamical figure of the Moon. Several analytical and numerical theories have been constructed over roughly the past 40 years (see Table 1), which have been used to achieve the precision required to implement lunar space projects and predict and describe many subtle effects in the Moon's rotation.

The series of the presented numerical ephemerides are the most complete, all have the same accuracy, and are adequate for modern radio technical and laser observations. A numerical approach makes it possible without particular mathematical problems to refine the model for the internal structure of the Moon

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Table 1. Theories describing physical libration of the Moon

Analytical description	Numerical ephemerides
Migus A. (1980)	
Eckhardt D. H. (1981; 1984)	
Moons M. (1982–1984)	DE 225, 403, 418, 421, 430/431, 432
Dubrovskii A., Chikanov Yu. (1987)	(JPL NASA, USA)
Petrova N. (1996)	EPM (ITA, then IAA, Russia)
Chapront J. et al. (1997; 1999)	INPOP (IMCCE, France)
Kondrat'ev B. (2013)	
Barkin Yu. et al. (2014)	
Empirical theory—Rambaux N., Williams J. (2011)	

and to take into account the required number of external perturbing factors, making this method flexible, precise, and satisfactory for good quality of modern observations. The situation is more complex when an analytical approach is used. For example, standard approaches based on Poisson series do not work directly if dissipative processes are included in the libration equations. The inclusion of high-order harmonics in the lunar potential and direct and indirect planetary perturbations makes it necessary to increase the dimensions of trigonometric and power-law indices, restructure analytical programs, incur a large expenditure of processor time, etc. Analytical theories are less accurate than numerical ones. Nevertheless, an analytical solution can enable a sort of "exploration" in terms of the manifestation of certain nuances in the lunar rotation, making it possible to distinguish markers in observations that are sensitive to these factors [13]. A harmonious combination of both approaches can enable progress in studies of the Moon.

A special place among theories of PhLM is occupied by the empirical theory of Rambaux and Williams [14], based on a computer analysis of residual differences computed during a comparison of the DE421 numerical ephemeris and laser-ranging data obtained during 1970–2007. A frequency analysis employing the method of Laskar [15] combined with a modified least-squares method was used to refine the frequencies and amplitudes of the harmonics of both forced and free librations. Such a study was first performed in 1996 by Chapront and Chapront-Touze [16]: based on the analytical theory of Moons [17], who calculated the frequencies of harmonics of the free libration and other harmonics produced by them for a rigid Moon, they processed data for the DE245 numerical theory and carried out a comparative anaysis in order to refine the frequencies of these harmonics, obtaining their amplitudes and phases for the first time.

The series of [14] were obtained, first, based on a direct comparison with observations, second, based on the more precise DE421 numerical theory, and third, applying modern computational techniques that appreciably expanded the possibilities of the method of [15]. Like the series of [16], these series describe so-called *wobble of the poles*, associated with the free Euler rotation of the Moon. In a lunar-dynamical coordinate frame, this manifests as direct motion of the rotational axis in an ellipse with axes of 3 and 8 arcsec and a period of 75 yrs; this is the long-period W mode, which produces several more harmonics in the motion of the poles when it interacts with forced librations. This behavior is described on the Earth as Chandler oscillations.

Two other free modes—*in longitude and latitude*—are not associated with the free rotation of the Moon; these arise against the background of the spin–orbital resonance in the Moon's motion and, according to the definition of Hansen, would more accurately be classified as *arbitrary* librations [18]. These modes are associated not with the motion of the body of the Moon relative to its rotation, but instead with a direct variation in space of its angular velocity in both magnitude and direction. The longitude mode U with a period of 2.9 yrs is similar to variations in the length of day, while the latitude mode V is similar to the nutation of the Earth. Due to the libration in latitude, the rotational axis of the Moon describes a small inverse cone in space with a period of 81 yrs.

The amplitudes of the harmonics corresponding to these two modes are very small, long making it impossible for them to be confidently detected in observations. In addition, the longitude harmonic U falls in a blend with several harmonics of forced librations due to the influence of Venus. Rambaux and Williams [14] were able to distinguish this low-amplitude harmonic only through complex modeling of the parameters of the libraton—its period, amplitude, and phase. As a result, the free libration in longitude was detected, and its amplitude at the initial epoch of JD 2000 was $\sim 1.3''$.

If we express the free libration in latitude V in terms of the libration of the node $I\sigma$ and the inclination ρ, this yields a period of $\sim$24 yrs. This mode most clearly manifests itself in the directional cosines of the ecliptic as the harmonic $F - V$ with an amplitude of 32 milliarcseconds (mas) and a period equal to a lunear sidereal month, 27.3 days. Since there are a large number of harmonics of forced librations at this resonance period for the Moon, it is far from straightforward to determine the parameters of free harmonics with the mode V.

The series [14] currently offer the most accurate analytical description of physical libration: they provide an accuracy of $0.04''$ in the libration inclination and node and $0.15''$ in the libration in latitude.

Very recently, Yang et al. [19] obtained refined parameters for *free modes for a rigid Moon* based on a comparison of a series of laser observations that is longer than [14] (by five years), using the currently most precise DE430 numerical ephemeris [12]. They present a compilation of data showing how the values of the studied parameters vary, beginning with the work of Chapront and Chapront-Touze [16] up to the current time, partially reflected in our Table 2.

3. MANIFESTATION OF A LIQUID CORE IN OBSERVATIONS OF PHYSICAL LIBRATION OF THE MOON

A no less important aspect of the empirical series that have been constructed is the presence of terms denoted Un, whose nature remains unknown. Most such terms are either long-period or have periods of about a day and have small amplitudes, of order several mas. The analytical theory of physical libration constructed by Barkin et al. [20] for a model of the Moon with an ellipsoidal, fluid core made it possible to identify the origin of these Un terms.

The first indirect evidence indicating that the Moon is not a completely rigid body, and possesses visco-elastic properties and a complex internal structure, was obtained from physical libration parameters derived via a reduction of laser-ranging data in the 1970s. Appreciable dissipation of the lunar rotation was detected at that time. The hypothesis that the origin of this dissipation was friction due to differential rotation of the rigid mantle and a fluid core was first proposed by Yoder [21]. Later, based on an analysis of a longer series of laser-ranging observatons, Williams et al. [5] concluded that the observed amplitudes of the libration harmonics that were sensitive to the dissipation of the rotation could only be explained by a combination of two factors: tidal friction and friction at the core–mantle boundary. Estimates of the size, aggregate state, and chemical composition of the core were obtained based on computer modeling of observations. These indirect signs of a fluid core were later confirmed in [2, 3] using direct methods based on modern techniques for reducing Apollo seismograms. As a result, our understanding of the structure of the central part of the Moon's body gradually became clearer: it contains a two-layered hot core—a rigid iron core with a diameter of $\sim$240 km and a fluid layer with a diameter of 330–360 km—surrounded by a partially molten shell with a diameter of $\sim$480 km.

Most analytical theories of PhLM have been constructed for models with a rigid Moon, although analytical estimates of the influence of the core on the Moon's rotation have been considered by many authors. In particular, Petrova et al. [18] showed that, if the Moon has a fluid core whose dynamical figure is similar to the mantle, an additional mode should appear in the motion of the lunar poles, with a period of 144–186 yrs, depending on the oblateness of the core. By analogy with the Earth, this harmonic is referred to as Free Core Nutation (FCN). According to the estimates of Barkin et al. [20], like the *fourth free mode of a non-rigid Moon* with an ellipsoidal, fluid core, FCN has a period of about 206 yrs.

The accuracy of gravimetric measurements made by the GRAIL lunar mission was insufficient to estimate the parameters of the core with the required accuracy [6]. The recent reductions of laser-ranging data [7, 19] likewise were not able to detect traces of the FCN harmonic: its period in inertial coordinates is very long, and in lunar-dynamical coordinates is close to a lunar sidereal month. It was proposed based on ambiguous data on the oblateness of the core that the FCN period should be more than 300 yrs [7].

The theory of Barkin et al. [20] was the first analytical *theory of forced and free librations for a two-layered model of the Moon* (with a fluid, ellipsoidal core). This theory incorporates a model core whose parameters—size, mass, moment of inertia—were estimated based on the data of [3, 6, 7, 14], as well as on gravimetric data obtained by the Selene mission.

This new theory opens a number of possibilities for estimating the parameters of the lunar core. It was shown by Barkin et al. [22] even earlier that the presence of a core will increase the amplitudes of *forced* harmonics of the librations, although at a level only 0.06% of the value for a rigid Moon. This small amplitude increase is determined mainly by the ratio of the moments of inertia of the core and the Moon as a whole, and to a lesser extent by the oblateness

Table 2. Parameters of free modes determined in various studies

Free modes	Period, days			Amplitude, $''$			Phase, deg		
	[14]	[19]	[20]	[14]	[19]	[20]	[14]	[19]	[20]
Longitude	1056.13	1056.16	1057.13	1.296	1.471	1.735	207.01	210.5	207.01
Latitude	8822.88	8806.9	8822.88	0.032	0.025	1.1881	160.81	160.67	160.81
Wobble	27 257.27 (74.6 yrs)	27 262.99 (74.6 yrs)	27 257.27 (74.6 yrs)	$3.306 \times$ $\times\, 8.183$	$3.19 \times$ $\times\, 8.31$	3.3072	161.60	161.64	161.60
Free core nutation (FCN)	71 954.25 (197 yrs) 3.8×10^{-4}	–	75 133.87 (205.7 yrs) 3.6×10^{-4}	–	–	0.0395	–	–	–134

The oblateness of the lunar core f_c is given in the last row.

of the core. Although this increase does not exceed $0.1''$ even for the most powerful harmonics of PhLM, it should be detectable in modern, highly accurate observations.

The hydrodynamical influence of the fluid core on the physical libration of the Moon was considered in [20] as a main factor capable of explaining terms in the empirical theory whose origins were unknown (Un). This study was aimed at searching for possible free librations due to an ellipsoidal, fluid core in series using the empirical theory for libration of the Moon [14] and identifying them using analytical series in which the frequencies of new librations are determined.

In this analysis, harmonics of analytically calculated free librations of the Moon for the first three modes (U, V, W) were clearly identified with the corresponding harmonics of the empirical theory, enabling determination of the amplitudes and initial phases of these three modes. More importantly, the harmonics of the lunar librations due to the fluid core could also be identified with similar terms in the unidentified Un librations.

A careful comparison of the analytical and corresponding empirical series made it possible to determine the parameters of the fourth mode of the free librations due to the fluid core for the first time (amplitude, initial phase for epoch JD 2000, and period; see Table 2). Further, it was possible to explain and interpret previously unidentified terms in the empirical theory. Eight harmonics of the free librations in the inclination ρ detected through an analysis of laser-ranging observations could be explained and interpreted mechanically as harmonics produced by FCN. A small free libration in longitude τ with a period of 7449.89^d and a low amplitude of $\sim 0.001''$ was also detected, likewise due to the influence of the ellipsoidal, fluid core.

As a result of studies of free librations of the Moon, the period $P_{FCN} = 75\,133.87^d$ and the known relationship between the oblateness of the core f_c and the period of the FCN, $P_{FCN} \sim \dfrac{27.3}{f_c}$ [13], were used as a basis to estimate the sum of the meriodional compressions, $\mu_D + \varepsilon_D = 7.244 \times 10^{-4}$. Based on the similarity of the dynamical compression ratios for the entire Moon and for the lunar core, the corresponding dynamical compressions of the core were found to be equal: $\varepsilon_D = \left(1 - \dfrac{A_c}{C_c}\right) = 4.42 \times 10^{-4}$ and $\mu_D = \left(1 - \dfrac{B_c}{C_c}\right) = 2.83 \times 10^{-4}$.

Thus, free librations due to the presence of an ellipsoidal, fluid core have been detected. The empirical series in which these harmonics were found are essentially *series of observations* presented in analytical form. This provides another direct argument demonstrating the existence of a fluid lunar core.

The effects of friction between the core and mantle were not considered in this model. The observational data testify to the presence of dissipation of the lunar rotation. It is already known from the analyses of laser-ranging data that a small fraction of the amplitudes of dissipative harmonics of PhLM is due to the influence of dissipative processes at the core–mantle boundary. The nature of these processes is also being discussed. Dissipation due to turbulent motions in the vicinity of the core–mantle boundary is also possible [4, 5, 21], as well as friction due to the viscosity of the fluid core. In this context, there is no doubt that the methods taking into account viscosity presented in [23, 24] are worthy of further attention. Nevertheless, at this stage of studies of the region of the core–mantle boundary, we have insufficient knowledge of the physical conditions in this zone and the numerical values of its parameters, such as temperature, viscosity, etc. Therefore, given the small influence of dissipation on PhLM, it is valid to use a model for the core without friction as a first approximation. The agreement between the observed frequencies and those calculated in the theory for a core without fraction again confirms the high degree of trustworthiness of the Barkin model with a two-layer Moon.

4. CONCLUSION

We have considered the role of observations and the theory of physical libration of the Moon to study the parameters of the internal structure of the Moon. Theoretical descriptions of physical librations are necessary in this process. The short review of available analytical and numerical theories we have presented shows the gradual development of this important area of lunar dynamics. Our multi-faceted approach has yielded information about the current visco-elastic properties of the Moon, demonstrated the presence of a small fluid core in the Moon, and identified new possibilities for refining the parameters of this core.

The results of the theory of Barkin et al. [20], constructed for a Moon with an ellipsoidal, fluid core, and their comparative analysis with the empirical series of [14] suggest the presence of *open free nutation of the core* of the Moon, and enable estimation of the parameters of this nutation and of the oblateness of the lunar core; we can now also speak of the observational detection of harmonics produced by the fourth free mode, in both the inclination ρ and the longitude τ, due to the hydrodynamical influence of the core on the Moon's rotation.

The important question of the sources of excitation of the detected free librations, which should decay with time, remains beyond the scope of this study. Hopes for the resolution of the question of the mechanisms supporting this excitation lie with new lunar experiments, including the Chinese mission Chang-E 3–4 and the Japanese ILOM experiment.

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